

# Coincidence problem in $f(T)$ gravity models

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## Abstract

It is well known fact that almost all the recent models of universe are plagued by the cosmic coincidence problem. In this assignment we try to probe the role played by torsion in the current scenario of coincidence and devise a set-up for its realization. In order to model the scenario, the energy arising from the torsion component is considered analogous to dark energy. An interaction between dark energy and dark matter is considered, which is by far the best possible tool to realize the coincidence. A set-up is designed and a constraint equation is obtained which screens the models of  $f(T)$  gravity that can successfully accommodate the stationary scenario in its framework, from those which cannot. Due to the absence of a universally accepted interaction term introduced by a fundamental theory, the study is conducted over three different forms of chosen interaction terms. As an illustration two widely known models of  $f(T)$  gravity are taken into consideration and used in the designed setup. The study reveals that the realization of the coincidence scenario as well as the role played by torsion in the current universe is a model dependent phenomenon. It is found that the first model showed a considerable departure from the stationary scenario. On the contrary the other four models are perfectly consistent with our setup and generated a satisfactory stationary scenario, thus showing their cosmological viability and their superiority over their counterparts. For the third model (exponential model) it was seen that the cosmological coincidence is realized only in the phantom regime. For the fourth (logarithmic model) and the fifth models, we see that the stationary scenario is attained for negative interaction values. This shows that the direction of flow must be from dark energy to dark matter unlike the previous models. Under such circumstances the universe will return from the present energy dominated phase to a matter dominated phase.

## 1 Introduction

At the turn of the last century the incompatibility of General Relativity (GR) came into light when cosmological observations from Ia supernovae, CMBR via WMAP, galaxy redshift surveys via SDSS indicated that the universe is going through an accelerated expansion of late (Perlmutter, S. et al. 1999; Spergel, D. N. et al. 2003; Bennett, C. L. et al. 2003; Tegmark, M. et al. 2004; Allen, S. W. et al. 2004). Since no possible explanation of this phenomenon could be given in the framework of Einstein's GR, a proper modification of the theory was required that will successfully incorporate the late cosmic acceleration. Two different approaches regarding this are widely found in literature.

Cosmic acceleration can be phenomenally attributed to the presence of a mysterious negative energy component popularly known as *dark energy (DE)* (Riess, A. G. et al. 2004). In this case the modification is brought about on the right hand side of the Einstein's equation, i.e. in the matter sector of the universe. The contribution of DE to the energy sector of the universe is  $\Omega_d = 0.7$ . In due course various candidates for DE began to appear in the scene. Some of the popular ones worth mentioning are Chaplygin gas models (Kamenshchik, A. et. al. 2001; Gorini, V. et. al. 2003), Quintessence Scalar field (Ratra, B. et. al. 1988), Phantom energy field (Caldwell, R. R. 2002), etc. All these models violate the strong energy condition i.e.,  $\rho + 3p < 0$ , thus producing the observed cosmic acceleration.

The alternative approach is based on the modification of the gravity sector of GR, thus giving birth to modified gravity theories. A universe associated with a tiny cosmological constant, i.e. the  $\Lambda$ CDM model served as a prototype for this concept. Although the model could satisfactorily explain the recent cosmic acceleration and passed a few solar system tests as well, yet, detailed diagnosis revealed that the model was paralyzed with a few cosmological problems. The two major problems that have crippled the model over the last decade are the Fine tuning problem (FTP) and the Cosmic Coincidence problem (CCP). The former refers to the large discrepancy between the observed values and the theoretically predicted values. There have been

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many attempts to solve this problem. The most impressive attempt was probably undertaken by Weinberg in (Weinberg, S. 1989). The solutions are basically based on the fact that the cosmological constant may not assume an extremely small static value at all times during the evolution of the universe, but its nature should be rather dynamical (Bisabr, Y. 2010). As a result alternative modifications of gravity was sought for. Some of the popular models of modified gravity that came into existence in recent times are loop quantum gravity (Rovelli, C. 1998; Ashtekar, A. et. al. 2004), Brane gravity (Brax, P. et. al 2004; Maartens, R. 2000, Maartens, R. 2004),  $f(R)$  gravity (Kerner, R. 1982; Allemandi, G. 2004; Carroll, S. M. et. al. 2005),  $f(T)$  gravity (Linder, E. V. 2010; Li, M. et. al. 2011; Miao, R. et. al. 2011; Li, B. et. al. 2011), etc. In recent times a lot of work has been done in modified gravity theories, specially  $f(R)$ ,  $f(T)$ ,  $f(R, T)$ , etc. models (Zubair, M. et. al. 2015a; Zubair, M. 2015b, Waheed, S. et. al. 2015; Harko, T. et. al. 2011; Sarkar, S. et. al. 2013; Sharif, M. et. al. 2013a, 2013b; Noureen, I. et. al. 2015).

In this work we will consider  $f(T)$  model as the theory of gravity.  $f(T)$  gravity is an alternative theory for GR, defined on the Weitzenbock non-Riemannian manifold. In this framework curvature is replaced by torsion. The formation is basically based on the division of the manifold into two separate but connected parts, one of which has a Riemannian structure with a definite metric and the other one has a non-Riemannian structure with torsion. The non-Riemannian part is based on a tetrad basis defining a Weitzenbock spacetime. The basis of this model was first laid by Einstein in his Teleparallel equivalent of general relativity (TEGR). At that time the purpose of the model was to unify electromagnetism and gravity. If we consider  $f(T) = T$ , the theory reduces to teleparallel gravity (Hayashi, K. et. al. 1979; Hehl, F. et. al. 1976). In (Hayashi, K. et. al. 1979), it has been shown that with a linear form  $f(T)$ , it is possible to satisfy the standard solar system tests, thus establishing the viability of the model. In spite of the success of the linear  $f(T)$  model, unfortunately further development took a long time coming. It was not before 2007 that Ferraro et al (Ferraro, R. et. al. 2007, 2008) introduced a general model of  $f(T)$  gravity. From then, there have been extensive work to enrich the theory. Here it is worth mentioning that Birkhoffs theorem was studied in this gravity by Meng et al (Meng, X. et. al. 2011). The authors in (Zheng, R. et. al. 2011) investigated perturbation in  $f(T)$  and found that the growth of perturbations in  $f(T)$  gravity is much slower than that in GR. Bamba et al (Bamba, K. et. al. 2011) studied the evolution of equation of state parameter and phantom crossing in  $f(T)$  model.

In (Bisabr, Y. 2010) Bisabr studied cosmological coincidence problem in the background of  $f(R)$  gravity. Motivated by Bisabr's work, we dedicate the present assignment to the study the effect of torsion arising from  $f(T)$  gravity models in the present coincidence scenario and devise a set-up for its realization. The paper is organized as follows: In section 2, the basic equations of  $f(T)$  gravity is presented. We address the coincidence problem in section 3 and the paper ends with some concluding remarks in section 4.

## 2 Basic Equations of $f(T)$ Gravity

A suitable form of action for  $f(T)$  gravity in Weitzenbock spacetime is (Ferraro, R. et. al. 2007, 2008)

$$S = \frac{1}{2\kappa^2} \int d^4x e [f(T) + L_m] \quad (1)$$

Here  $e = \det(e_\mu^i) = \sqrt{-g}$ ,  $\kappa^2 = 8\pi G$  and  $e_\mu^i$  is the tetrad (vierbein) basis. The dynamical quantity of the model is the vierbein  $e_\mu^i$  and  $L_m$  is the matter Lagrangian. Taking the variation of the action (1), with respect to the vierbein  $e_\mu^i$ , the modified Friedmann equations in the spatially flat FRW universe can be obtained as,

$$H^2 = \frac{\kappa^2}{3} (\rho_m + \rho_T) \quad (2)$$

$$2\dot{H} + 3H^2 = -\kappa^2 (p_m + p_T) \quad (3)$$

where

$$\rho_T = \frac{1}{2\kappa^2} (2T f_T - f - T) \quad (4)$$

$$p_T = -\frac{1}{2\kappa^2} \left[ -8T\dot{H}f_{TT} + (2T - 4\dot{H}) f_T - f + 4\dot{H} - T \right] \quad (5)$$

and

$$T = -6H^2 \quad (6)$$

Here the subscript  $T$  indicates derivative with respect to the torsion scalar  $T$  and obviously  $H$  denotes the Hubble parameter.  $\rho_m$  and  $p_m$  represents the energy density and pressure of the matter content of the universe whereas  $\rho_T$  and  $p_T$  represents the density and pressure contributions of the scalar torsion.

The energy conservation equations are given by,

$$\dot{\rho}_m + 3H\rho_m = Q \quad (7)$$

$$\dot{\rho}_T + 3H(1 + \omega_T)\rho_T = -Q \quad (8)$$

Here  $\omega_T = \frac{p_T}{\rho_T}$  is the EoS parameter of the torsion scalar and  $Q$  is the interaction between the matter and the torsion sector of the universe.

The torsion EoS parameter is defined as (Wu, P. et. al. 2010, Karami, K. et. al. 2012)

$$\omega_T = \frac{p_T}{\rho_T} = -1 - \frac{\dot{T}}{3H} \left( \frac{2Tf_{TT} + f_T - 1}{2Tf_T - f - T} \right) \quad (9)$$

For a de-Sitter universe (empty),  $\dot{H} = \dot{T} = 0$ . So eqn.(9) gives  $\omega_T = -1$ , which corresponds to the  $\Lambda$ CDM model.

Using eqns.(2), (4) and (6), we calculate the matter density as,

$$\rho_m = \frac{1}{2\kappa^2} (f - 2Tf_T) \quad (10)$$

Here we will consider pressureless matter. So using  $p_m = 0$  in the eqn.(3) and then using eqns.(2) (5) and (9) we obtain,

$$\dot{T} = 3H \left( \frac{f - 2Tf_T}{2Tf_{TT} + f_T} \right) \quad (11)$$

Using the above expression of  $\dot{T}$  in eqn. (9), we arrive at the final expression for EoS parameter for torsion scalar as,

$$\omega_T = -\frac{f/T - f_T + 2Tf_{TT}}{(f_T + 2Tf_{TT})(f/T - 2f_T + 1)} \quad (12)$$

The deceleration parameter is given by

$$q = -1 - \frac{\dot{H}}{H^2} \quad (13)$$

Using eqns.(6) and (11), we get the expression for deceleration parameter in  $f(T)$  gravity as,

$$q = 2 \left( \frac{f_T - Tf_{TT} - 3f/4T}{f_T + 2Tf_{TT}} \right) \quad (14)$$

For teleparallel equivalent of general relativity (TEGR),  $f(T) = T$ . Using this in the above equation we get  $q = 0.5$ , which corresponds to decelerating universe, which in turn points towards a matter dominated scenario.

### 3 The Cosmic Coincidence Problem

The cosmic coincidence problem has been a serious issue in recent times regarding various dark energy models. Recent cosmological observations have shown that densities of the matter sector and the DE sector of the universe are almost identical in the late universe. We know that the matter and the energy component of the universe have evolved independently from different mass scales in the early universe, then how come they reconcile to the same mass scales in the late universe! This is our problem. Almost all the DE models known till date suffer from this phenomenon.

There have been numerous attempts to solve the coincidence problem. Among them the most impressive one is the concept of a suitable interaction between the matter and the energy components of the universe, as used in the conservation equations (7) and (8). Here we consider that the two sectors of the universe have not evolved independently from different mass scales. But they interact with each other, thus allowing a mutual flow between the two components. As a result, the densities of the two components coincide in the present universe. Although the concept seems to be promising yet there remains a problem. There is no universally accepted interaction term, introduced by a fundamental theory.

### 3.1 Choice of Interaction term

Due to the unknown nature of both dark energy and dark matter, it is not possible to derive an expression for interaction ( $Q$ ) using the first principles. So in such a situation, one is expected to use logical reasoning and propose various feasible expressions for  $Q$ . Observing the domination of dark energy in late times the best possible argument is to consider  $Q$  to be small and positive. A large negative  $Q$  will see the universe dominated by dark energy from the early times, thus leaving no scope for the condensation of galaxies. The most obvious choice for interaction should be the energy densities multiplied by the hubble parameter, because it is both physically and dimensionally justified. So  $Q = Q(H\rho_m, H\rho_{de})$ , where  $\rho_{de}$  is the dark energy density. Since here we are not adding any dark energy by hand, so the torsion component of energy,  $\rho_T$  will replace  $\rho_{de}$ . This leads us to three basic forms of interactions as given below (del Campo, S. et. al. 2009):

$$\Gamma - model : Q = 3\Gamma H (\rho_m + \rho_T), \quad \zeta - model : Q = 3\zeta H \rho_m, \quad \eta - model : Q = 3\eta H \rho_T \quad (15)$$

where  $\Gamma$ ,  $\zeta$  and  $\eta$  are the coupling parameters of the respective interaction models.

It is worth mentioning that due to its simplicity the most widely used interaction model is the  $\zeta$ -model and is available widely in literature (Berger, M. S. et. al. 2006, del Campo, S. et. al. 2009, Rudra, P. et. al. 2012, Jamil1).

### 3.2 Coincidence in presence of torsion: The set-up

In this note we try to probe the role played by the torsion component of the gravity theory in producing the present coincidence scenario.  $f(T)$  gravity has evolved over the past few years as a major candidate of modified gravity theory satisfying all the solar system tests.  $f(T)$  gravity is itself self competent in producing the late cosmic acceleration without resorting to any forms of dark energy. Therefore we do not consider any separate dark energy components in the present study, but the equivalent energy evolving from the torsion component of the  $f(T)$  gravity is considered as the dark energy. We consider the ratio of the densities of matter and dark energy as,  $r \equiv \rho_m/\rho_T$ . Our aim is to search for some appropriate forms of the function  $f(T)$  that produces a stationary value of the ratio of the component densities,  $r$ . The time evolution of  $r$  is as follows,

$$\dot{r} = \frac{\dot{\rho}_m}{\rho_T} - r \frac{\dot{\rho}_T}{\rho_T} \quad (16)$$

Using eqns. (7), (8) and (16), we obtain

$$\dot{r} = 3Hr\omega_T + \frac{Q}{\rho_T} (1+r) \quad (17)$$

Using eqns. (4), (12) and (14) in eqn.(17), we get,

$$\dot{r} = 3Hr \left[ \frac{3r}{2(r+1)(-12H^2 f_{TT} + f_T)} - \left( \frac{f + 12H^2 f_T}{-12H^2 f_{TT} + f_T} \right) \left( \frac{-12H^2 f_{TT} + f_T - 1}{-12H^2 f_T - f + 12H^2} \right) + q \right] + \frac{2\kappa^2 (1+r) Q}{-12H^2 f_T - f + 6H^2} \quad (18)$$

Now in order to comply with observations, it is required that universe should approach a stationary stage, where either  $r$  becomes a constant or evolves slower than the scale factor. In order to satisfy this  $\dot{r} = 0$  in the present epoch. It leads to the following equation,

$$g(f, H, r_s, q) = 0 \quad (19)$$

where

$$g(f, H, r_s, q) \equiv 3Hr_s \left[ \frac{3r_s}{2(r_s+1)(-12H^2 f_{TT} + f_T)} - \left( \frac{f + 12H^2 f_T}{-12H^2 f_{TT} + f_T} \right) \left( \frac{-12H^2 f_{TT} + f_T - 1}{-12H^2 f_T - f + 12H^2} \right) + q \right] + \frac{2\kappa^2 (1+r_s) Q}{-12H^2 f_T - f + 6H^2} \quad (20)$$

where  $r_s$  is the value of  $r$  when it takes a stationary value.

Now eqn.(19) gives us a constraint which can be used to find suitable  $f(T)$  functions consistent with a late time stationary scenario of energy densities. It can also be used to check whether a particular  $f(T)$  model fits the

stationary scenario or not. For a given redshift  $z_0$  at a sufficiently late time, the corresponding contemporary parameters will be given by  $r_s(z_0)$ ,  $H(z_0)$  and  $q(z_0)$ . At sufficiently late times eqn.(19) can be rewritten as,

$$g(f_0, H_0, r_{s0}, q_0) = 0 \quad (21)$$

where

$$g(f_0, H_0, r_{s0}, q_0) \equiv 3H_0 r_{s0} \left[ \frac{3r_{s0}}{2(r_{s0} + 1)(-12H_0^2 f_0'' + f_0')} - \left( \frac{f_0 + 12H_0^2 f_0'}{-12H_0^2 f_0'' + f_0'} \right) \left( \frac{-12H_0^2 f_0'' + f_0' - 1}{-12H_0^2 f_0' - f_0 + 12H_0^2} \right) + q \right] + \frac{2\kappa^2(1 + r_{s0})Q}{-12H_0^2 f_0' - f_0 + 6H_0^2} \quad (22)$$

where  $r_{s0} = r_s(z_0)$  and  $f_0, f_0', f_0''$  represents the late-time configurations of  $f, f_T$  and  $f_{TT}$ .

As far as  $q$  is concerned, we start from the best fit parametrization obtained directly from observational data. Here we use a two parameter reconstruction function for  $q(z)$  (Gong, Y. G. et. al. 2006, 2007)

$$q(z) = \frac{1}{2} + \frac{q_1 z + q_2}{(1 + z)^2} \quad (23)$$

On fitting this model to Gold data set, we get  $q_1 = 1.47^{+1.89}_{-1.82}$  and  $q_2 = -1.46 \pm 0.43$  (Gong, Y. G. et. al. 2007). We consider  $z_0 = 0.25$  and using these values in eqn.(23), we get  $q_0 \approx -0.2$ . From recent observations, we obtain  $r_0 \equiv \frac{\rho_m(z_0)}{\rho_T(z_0)} \approx \frac{3}{7}$  (Zlatev, I. et. al. 2000; Wei, H. et. al. 2005; Yang, G. et. al. 2005).

Now in order to illustrate the present scheme of work we will consider some specific models and use them in the constraint eqn.(21). The models that will satisfy the constraint equation will be cosmologically more acceptable models since they admit the cosmic coincidence in their framework. In the light of this, we may possibly be able to rule out some of the known models which will not satisfy the constraint, thus becoming inconsistent with observations.

### 3.3 Illustration

#### 3.3.1 Model 1

This model was proposed by Abdalla et al in 2005 (Abdalla, M. C. B. et. al. 2005). It is given by,

$$f(T) = \beta T + \gamma T^n \quad (24)$$

where  $\beta, \gamma$  and  $n > 0$  is a constants.

We fit the above model in the constraint eqn.(21) and plot the function  $g(f_0, H_0, r_{s0}, q_0)$  against the parameter  $n$  in figs. (1a), (1b) and (1c) for  $\Gamma, \zeta$  and  $\eta$  interaction terms respectively. From the figures it is seen that the function  $g$  never attains the zero level. So the constraint (21) is not satisfied for any values of  $n$ . Hence the model does not admit a late time stationary ratio of energy densities. Therefore it is not a cosmologically viable model as far as the latest observational data is concerned.

#### 3.3.2 Model 2

Our second model is the model proposed in (Jamil, M. et. al. 2012). In this model, to avoid analytic and computation problems, we choose a suitable expression for  $f(T)$  which contains a constant, linear and a non-linear form of torsion. The model is given by

$$f(T) = 2C_1 \sqrt{-T} + \alpha T + C_2, \quad (25)$$

where  $\alpha, C_1$  and  $C_2$  are arbitrary constants. It is evident from the model that  $C_1 = 0$  leads to Teleparallel gravity. The combination of the first and the third term of the model corresponds to the cosmological constant EoS in the background of  $f(T)$  gravity (Myrzakulov, R. 2011). Likewise by shuffling terms, cosmologists have been able to set up various models of  $f(T)$  gravity possessing distinguishable features. It is worth mentioning the model that we are dealing with presently may have gathered its motivation from the model of Veneziano ghost (Karami, K. et. al. 2013)

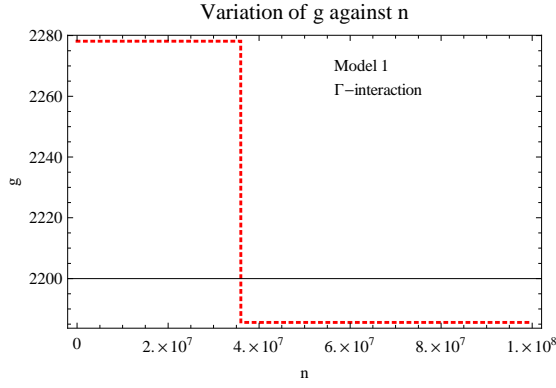


Fig.1a

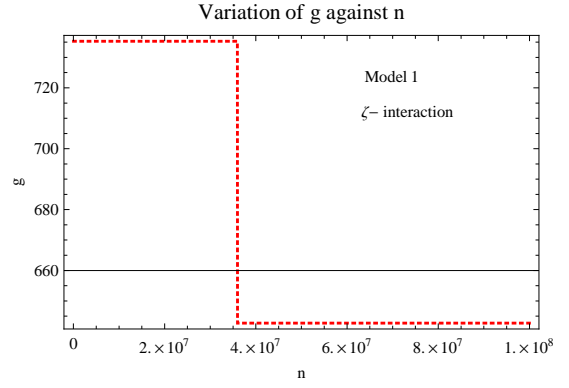


Fig.1b

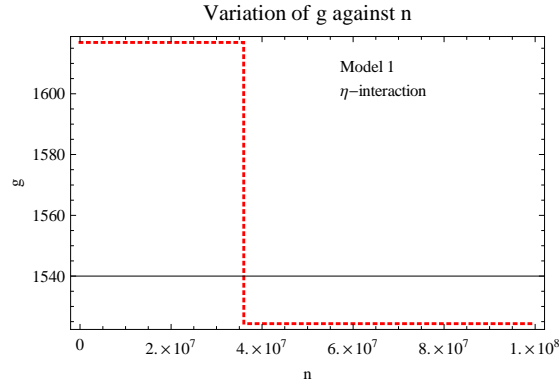


Fig.1c

Fig 1a : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $n$  for model 1 using  $\Gamma$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \beta = 0.1, \gamma = 0.4, \Gamma = 0.5$

Fig 1b : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $n$  for model 1 using  $\zeta$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \beta = 0.1, \gamma = 0.4, \zeta = 0.5$

Fig 1c : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $n$  for model 1 using  $\eta$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \beta = 0.1, \gamma = 0.4, \eta = 0.5$

There are two basic advantages of this model as a result of which it is preferentially chosen over other alternatives. The first one is its simplicity with numerical computations and other one is the results obtained from this model can be easily compared to the corresponding results in general relativity. An analysis performed by Capozziello et al in (Capozziello, S. et. al. 2011) showed that by choosing  $C_1 = \sqrt{6}H_0(\Omega_{m0} - 1)$ ,  $C_2 = 0$  and  $\alpha = \Omega_{m0}$ , it is possible to estimate the parameters of the model as functions of Hubble parameter, cosmographic parameters and matter density parameters. Here  $\Omega_{m0} = \frac{\rho_{m0}}{3H_0^2}$  represents the present value of dimensionless matter density parameter. In accordance with the current observational data  $\Omega_{m0} = 0.3$ .

We fit this model in the constraint eqn.(21) and plot the function  $g(f_0, H_0, r_{s0}, q_0)$  against the parameter  $\alpha$  in figs.(2a), (2b) and (2c) for  $\Gamma$ ,  $\zeta$  and  $\eta$  interaction terms respectively. From figure (2a) it is evident that the constraint (21) is satisfied for  $\alpha \approx -0.5$ . Similarly in figs. (2b) and (2c), the constraint is satisfied for  $\alpha \approx -1$  and  $\alpha \approx -0.6$  respectively. This implies that for these values of  $\alpha$ , the model admits a late time stationary scenario as far as the ratio of energy densities is concerned. Therefore in the background of this model, and considering torsion contribution to be analogous to dark energy a stationary scenario is successfully achieved for various interaction terms. Hence this model is perfectly viable according to the latest cosmological observations.

### 3.3.3 Model 3 (Exponential Model)

The model is given by (Linder, E. V. 2010; Bamba, K. et al 2011)

$$f(T) = \alpha_1 T \left(1 - e^{p \frac{T_0}{T}}\right) \quad (26)$$

where  $\alpha_1 = -\frac{1-\Omega_m^{(0)}}{1-(1-2p)e^p}$  and  $p$  is a constant. It must be noted that  $p = 0$  corresponds to the  $\Lambda$ CDM model. Moreover  $T_0 = T(z = 0)$  is the torsion in the present time. Here  $\Omega_m^{(0)} = \rho_m^{(0)}/\rho_{crit}^{(0)}$ , where  $\rho_m^{(0)}$  is the energy density of non-relativistic matter in the present time and  $\rho_{crit}^{(0)} = \frac{3H_0^2}{8\pi G}$  is the critical density. It is to be noted that the parameter  $\alpha_1$  that appears in the exponential model, has been derived from  $\Omega_{DE}^{(0)} = \rho_{DE}^{(0)}/\rho_{crit}^{(0)} = 1 - \Omega_m^{(0)} = -\alpha [1 - (1 - 2p)e^p]$  using the energy density of the effective dark energy of  $f(T)$  theory. If  $\Omega_m^{(0)}$  is given, then the exponential model has a single parameter  $p$ . Using  $p$  and  $\Omega_m^{(0)}$  we can calculate the values of other dimensionless quantities at present time ( $z = 0$ ). In (Bamba, K. et al 2011), we see that for  $p > 0$ , the universe enters the phantom regime, whereas for  $p < 0$ , the universe remains in the quintessence era.

We fit this model in the constraint eqn.(21) and plot the function  $g(f_0, H_0, r_{s0}, q_0)$  against the parameter  $p$  in figs.(3a), (3b) and (3c) for  $\Gamma$ ,  $\zeta$  and  $\eta$  interaction terms respectively. From the figures it is seen that for all the three interaction models the stationary scenario is realized as far as the dark energy and dark matter is concerned. For  $\Gamma$ ,  $\zeta$  and  $\eta$  interaction models, stationary scenario is realized for  $p = 0.15, p = 0.30$  and  $p = 0.42$  respectively. From the figures it is clear that the coincidence scenario is obtained only for positive values of  $p$ , i.e., when the universe enters the phantom regime.

### 3.3.4 Model 4 (Logarithmic Model)

The Logarithmic model of  $f(T)$  theory is given by (Bamba, K. et al 2011)

$$f(T) = \beta_1 T_0 \left(\frac{\epsilon T_0}{T}\right)^{-1/2} \ln \left(\frac{\epsilon T_0}{T}\right) \quad (27)$$

where  $\beta_1 \equiv \frac{1-\Omega_m^{(0)}}{2q^{-1/2}}$  and  $\epsilon$  is a positive constant. It must be noted that if  $\Omega_m^{(0)}$  is obtained in the same way as in the exponential  $f(T)$  model, then  $\epsilon > 0$  is the only parameter of the logarithmic model. It was found in (Bamba, K. et al 2011) that equation of state ( $\omega$ ) never crosses the phantom line for this model.

On fitting this model in the constraint eqn.(21) we get the corresponding plot for the function  $g(f_0, H_0, r_{s0}, q_0)$  against the parameter  $\epsilon$  in figs.(4a), (4b) and (4c) for  $\Gamma$ ,  $\zeta$  and  $\eta$  interaction terms respectively. From the figures it is seen that for all the three interaction models the coincidence scenario is realized between dark energy and dark matter. For  $\Gamma$ ,  $\zeta$  and  $\eta$  interaction models, stationary scenario is realized for  $\epsilon = 3950, \epsilon = 4670$  and  $\epsilon = 2800$  respectively. It must be noted here that stationary scenario in this case is realized only for negative value of interaction coupling constant. This shows that the direction of flow is just the reverse compared to the previous models, i.e., from dark energy to dark matter.

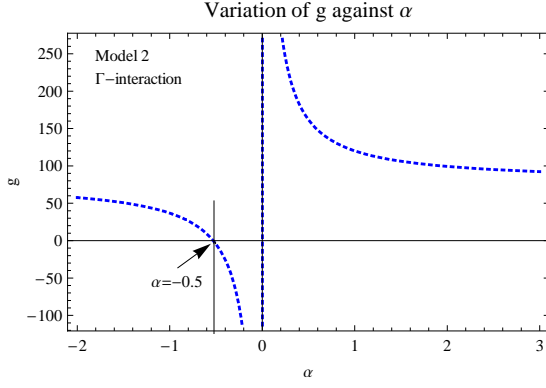


Fig.2a

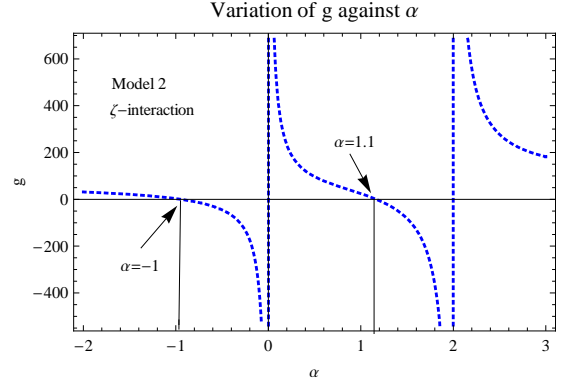


Fig.2b

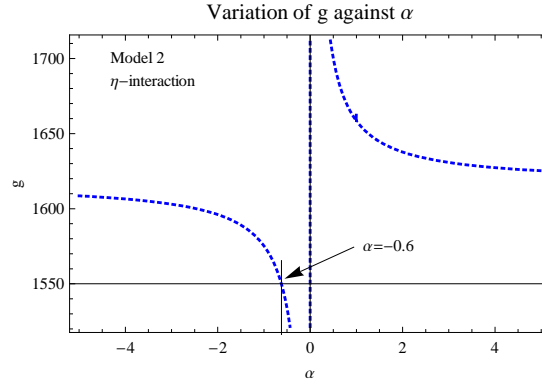


Fig.2c

Fig 2a : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $\alpha$  for model 2 using  $\Gamma$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \Gamma = 0.01, C_1 = \sqrt{6}H_0 (\Omega_{m0} - 1) = -123.4542830, C_2 = 0$

Fig 2b : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $\alpha$  for model 2 using  $\zeta$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, C_1 = \sqrt{6}H_0 (\Omega_{m0} - 1) = -123.4542830, C_2 = 0, \zeta = 0.5$ .

Fig 2c : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $\alpha$  for model 2 using  $\eta$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, C_1 = \sqrt{6}H_0 (\Omega_{m0} - 1) = -123.4542830, C_2 = 0, \eta = 0.5$



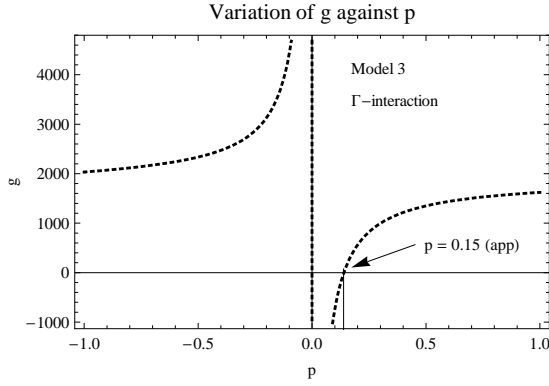


Fig.3a

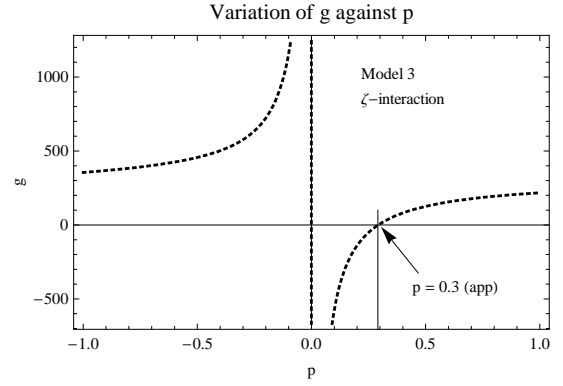


Fig.3b

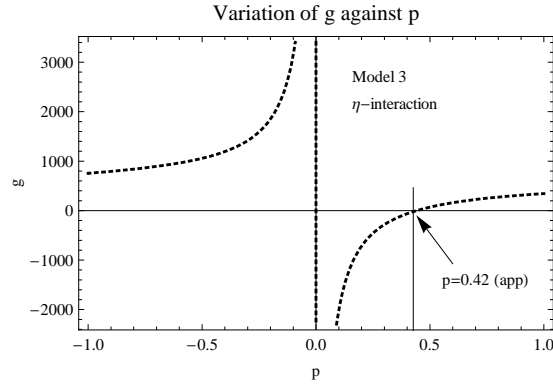


Fig.3c

Fig 3a : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $p$  for model 3 using  $\Gamma$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \Gamma = 5$ .

Fig 3b : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $p$  for model 3 using  $\zeta$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \zeta = 3$ .

Fig 3c : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $p$  for model 3 using  $\eta$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \eta = 3$ .

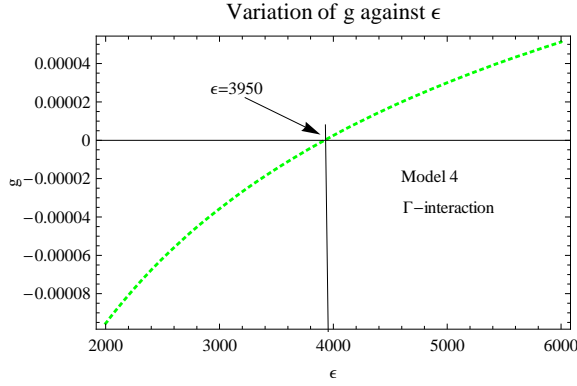


Fig.4a

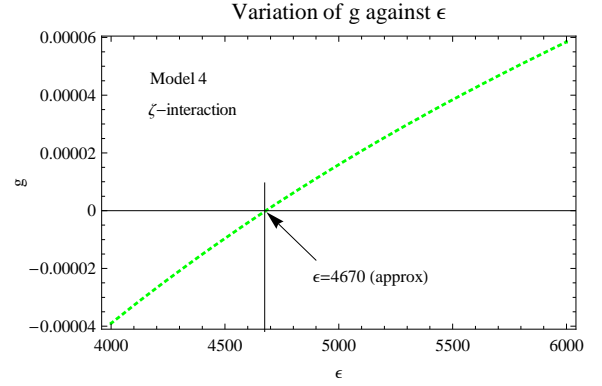


Fig.4b

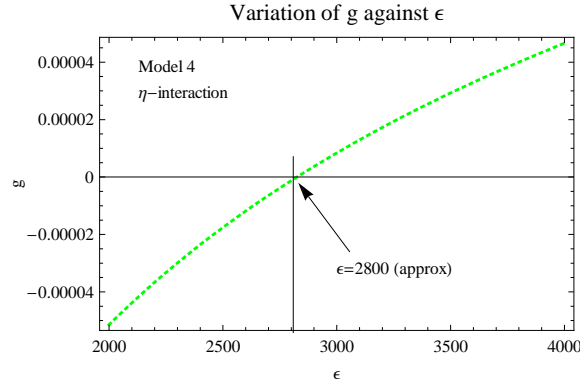


Fig.4c

Fig 4a : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $T/T_0$  for model 4 using  $\Gamma$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \Gamma = -.167998$

Fig 4b : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $T/T_0$  for model 4 using  $\zeta$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \zeta = -.559987$ .

Fig 4c : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $T/T_0$  for model 4 using  $\eta$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \eta = -.239997$ .

### 3.3.5 Model 5

The model is given by (Myrzakulov, R. 2011)

$$f(T) = \alpha_2 T + \frac{\beta_2}{T} \quad (28)$$

where  $\alpha_2$  and  $\beta_2$  are constants.

Here also we see that the constraint eqn.(21) is satisfied and a satisfactory stationary scenario is achieved between dark energy and dark matter. From the figs. (5a), (5b) and (5c), we see that the coincidence is attained for all the three interaction models  $\Gamma$ ,  $\zeta$  and  $\eta$  at  $\alpha_2 = 950$ ,  $\alpha_2 = 1015$  and  $\alpha_2 = 1350$  respectively. Just like the previous model, we see that in this model also a negative value of coupling constant is necessary to attain the stationary scenario. Therefore, here also the direction of flow is from dark energy to dark matter.

## 4 Conclusion

In this assignment we have devised a method to probe the contribution of torsion, interacting with dark matter to realize the present scenario of cosmic coincidence. The comparable values of the densities of dark energy (energy from torsion component) and dark matter have been attributed to the presence of a suitable interaction between them. The choice of the interaction term obviously was not unanimous. So we decided to consider three different forms of possible interaction terms widely found in literature. A setup was designed and a constraint equation was formed which will filter the models that accommodate a stationary scenario in its framework, from those which do not. This will help us to differentiate the cosmologically viable models from the others. To demonstrate the designed set-up, we considered five specific models of  $f(T)$  gravity available in literature, using all the three forms of interactions. It was found that the first model showed a considerable departure from the stationary scenario. On the contrary the other four models were perfectly consistent with our setup and generated a satisfactory stationary scenario, thus showing their cosmological viability and their superiority over their counterparts. For the third model (exponential model) it was seen that the cosmological coincidence is realized only in the phantom regime, which is a very interesting result. For the fourth (logarithmic model) and the fifth models, we see that the stationary scenario is attained for negative interaction values. This shows that the direction of flow must be from dark energy to dark matter unlike the previous models. Under such circumstances the universe will return from the present energy dominated phase to a matter dominated phase. It can also be speculated that this phenomenon may analogically correspond to a future deceleration (Chakraborty et. al. 2014; Pan et. al. 2015) or even a cosmic contraction. This result is quite unexpected. The reason behind this is not quite clear and we keep it an open question for the time being. Moreover, it must be mentioned that the realization of the stationary scenario for the fourth and the fifth models was quite restrained. From the numerical simulations it was found that the scenario could only be achieved for the given values of the interaction coupling constants. So its clear that the coincidence phenomenon actually constrains the interaction parameters for these models.

## Acknowledgements

The Author acknowledges the anonymous referee for enlightening comments that helped to improve the quality of the manuscript.

## Compliance with Ethical Standards

The author declares that he has no conflict of interest.

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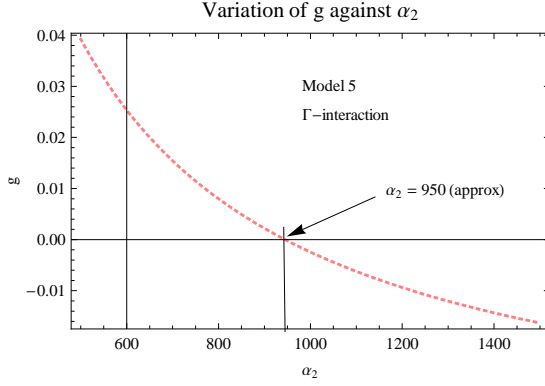
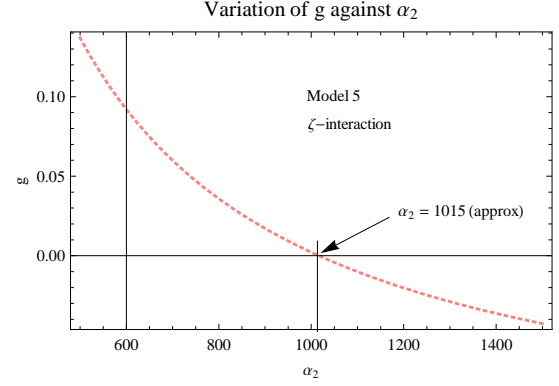


Fig.5a



5b

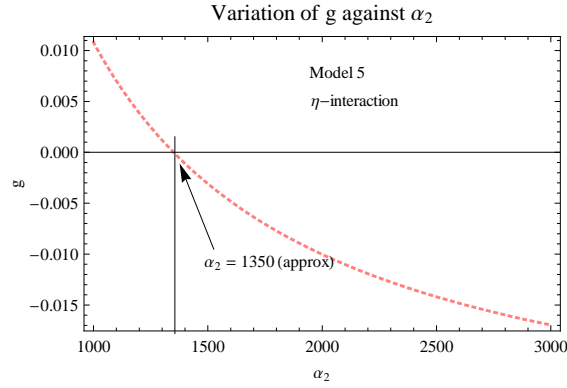


Fig.5c

Fig 5a : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $\alpha_2$  for model 5 using  $\Gamma$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \beta = 2, \Gamma = -0.1681$ .

Fig 5b : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $\alpha_2$  for model 5 using  $\zeta$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \beta = 2, \zeta = -0.561$

Fig 5c : The plot of  $g(f_0, H_0, r_{s0}, q_0)$  against  $\alpha_2$  for model 5 using  $\eta$  interaction. The other parameters are considered as  $q = -0.2, r = 3/7, H_0 = 72, \beta = 2, \eta = -0.2401$ .

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